Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Real Analysis-I

Subject Code : 5SC02MTC4 **Branch: M.Sc (Mathematics)**

Semester : 2 **Date 11/05/2016** **Time : 10:30 To 1:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions .** **(07)**

a. What is cluster point of sequence ? **(02)**
b. Define : limit inf of sequence. **(02)**
c. Define: Borel set **(02)**
d. What is outer measure of Q ? **(01)**

Q-2 **Attempt all questions** **(14)**

a. Show that the outer mesure of an interval is its length. **(07)**
b. What is measurable set? If E_1 and E_2 are two measurable set then show that $E_1 \cup E_2$ is also measurable. **(07)**

OR

Q-2 **Attempt all questions** **(14)**

a. What is measurable function ? If f and g are measurable then show that f + g is also measurable function. **(07)**
b. Show that the collection $\mathfrak{m}$ of all measurable set is a σ-algebra. **(07)**

Q-3 **Attempt all questions** **(14)**

a. Let φ and ψ be simple functions define on set of finite measure E ,then show that for any α and β $\int_E (\alpha\varphi + \beta\,\psi) = \alpha\int_E \varphi + \beta\int_E \psi$. **(07)**

b. What is lebsege integrable function ? if f is bounded measurable function on a set of finite measure E .Suppose A and B are disjoint measurable subset of E then show that **(07)**

(I) $\int_{A\cup B} f = \int_A f + \int_B f$.

(II) $\left|\int_E f\right| \le \int_E |f|$

OR

Q-3 **a.** State and prove bounded convergence theorem. **(07)**

b. State and prove fatau's lemma. **(07)**

SECTION – II

Q-4 **Attempt the Following questions .** **(07)**

a. What is BV[a,b] ? **(02)**

b. Is the point wise limit of sequence of measurable functions measurable ? justify your answer . **(02)**

c. What are f(x)= sinx then what is $f^{+}(x)$? **(02)**

d. Define : g_δ-set . **(01)**

Q-5 **Attempt all questions** **(14)**

a. State and prove lebsege dominated convergence theorem. **(07)**

b. State and prove Beppo -levis theorem **(07)**

OR

Q-5 **a.** State and prove Jordan's lemma. **(07)**

b. State and prove Fundamental theorem of integral calculus. **(07)**

Q-6 **Attempt all questions** **(14)**

a. Write Littlewood's three principles. **(03)**

b. Apply BCT to Evaluate the following lebsege integral **(04)**

$$\lim_{n\to\infty} \int_{[2,10]} \frac{nx}{1+n^2x^2}\,dx$$

c. Suppose f and g are in BV[a,b] , then show that f+g and f.g are in BV[a,b] **(07)**

OR

Q-6 **Attempt all Questions**

a. If f is measurable function then show that f + c is also measurable. **(03)**

b. What are the positive and negative variations of function f(x). **(04)**

c. Suppose f is integrable on [a,b] , let $\int_a^x f(t)dt = 0$; $x \in [a,b]$ then show that f = 0 a.e. **(07)**